Find the general solution of $y^{(4)} - 2y'' + y = 3e^x$.

SCORE: ____ / 14 PTS

$r^4 - 2r^2 + 1 = 0$

$(r^2 - 1)^2 = 0$

$(r+1)^2(r-1)^2 = 0$

$r = 1, 1, -1, -1$

$y_h = Ae^x + Bxe^x + Ce^{-x} + Dxe^{-x}$

(3) $y_p = Kx^2e^x$

$y_p' = (Kx^2 + 2Kx)e^x$

$y_p'' = (Kx^2 + 4Kx + 2K)e^x$

$y_p''' = (Kx^2 + 6Kx + 6K)e^x$

$y_p^{(4)} = (Kx^2 + 8Kx + 12K)e^x$

$y_p^{(4)} - 2y_p'' + y_p = (Kx^2 + 8Kx + 12K$
$-2Kx^2 - 8Kx - 4K$
$+ Kx^2)e^x$

$= 8Ke^x = 3e^x$ (2)

$8K = 3$

$K = \frac{3}{8}$

$y = \frac{3}{8}x^2e^x + Ae^x + Bxe^x + Ce^{-x} + Dxe^{-x}$

Find the general solution of $y''+4y'+4y=\dfrac{1}{(1+x^2)e^{2x}}$.

SCORE: ____ / 10 PTS

$r^2+4r+4=0$

$(r+2)^2=0$

$r=-2,-2$

$y_1=e^{-2x}, y_2=xe^{-2x}$

$$W=\begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x}-2xe^{-2x} \end{vmatrix} = e^{-4x}$$

$$y_p=-e^{-2x}\int \frac{1}{(1+x^2)e^{2x}}\frac{xe^{-2x}}{e^{-4x}}\,dx + xe^{-2x}\int \frac{1}{(1+x^2)e^{2x}}\frac{e^{-2x}}{e^{-4x}}\,dx$$

$$=-e^{-2x}\int \frac{x}{1+x^2}\,dx\ (\tfrac{1}{2}) + xe^{-2x}\int \frac{1}{1+x^2}\,dx\ (\tfrac{1}{2})$$

$$=-e^{-2x}\left(\tfrac{1}{2}\ln(1+x^2)\right)+xe^{-2x}\tan^{-1}x$$

$$y=-\tfrac{1}{2}e^{-2x}\ln(1+x^2)+xe^{-2x}\tan^{-1}x+Ae^{-2x}+Bxe^{-2x}$$

Use the annihilator method to find the form of a particular solution of $y''-10y'+29y=(1+e^{5x})\sin 2x$.

SCORE: ____ / 6 PTS

You MUST show the usage of the annilator. Do NOT solve for the coefficients in the particular solution.

$r^2-10r+29=0$

$(r-5)^2+4=0$

$r=5\pm 2i$

$y_h=Ae^{5x}\cos 2x+Be^{5x}\sin 2x$

$(1+e^{5x})\sin 2x=\sin 2x+e^{5x}\sin 2x$

$r=\pm 2i, 5\pm 2i$

$A=(D^2+4)((D-5)^2+4)$ (2)

$(D^2+4)((D-5)^2+4)(D-5)^2+4)(y)=(D^2+4)(D-5)^2+4)((1+e^{5x})\sin 2x$

$(D^2+4)((D-5)^2+4)^2(y)=0$

$r=\pm 2i, 5\pm 2i, 5\pm 2i$

$y=A\cos 2x+B\sin 2x+Ce^{5x}\cos 2x+De^{5x}\sin 2x+Fxe^{5x}\cos 2x+Gxe^{5x}\sin 2x$

$y_p=A\cos 2x+B\sin 2x+Fxe^{5x}\cos 2x+Gxe^{5x}\sin 2x$